RELATIONAL VERSUS INSTRUMENTAL UNDERSTANDING IN BASIC SCHOOL MATHEMATICS: INSIGHTS FROM SKEMP'S THEORY OF COGNITIVE DEVELOPMENT AND ITS IMPLICATIONS FOR PRACTICAL TEACHING

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ABSTRACT

Mathematics learning at the basic school (primary and lower secondary) level often suffers from persistent issues: students learn procedures without understanding, struggle with applying knowledge to novel tasks, and show weak retention and transfer. In his foundational work, Skemp (1976, 1987) introduced the terms instrumental and relational understanding to capture two qualitatively different modes of mathematical cognition. This paper revisits Skemp's theory in the context of basic school mathematics, synthesises recent empirical work (2022-2025) that applies or extends his ideas, and proposes implications for curriculum, teaching, assessment and teacher professional development. We argue that to foster meaningful, transferable and enduring mathematical learning for young learners, instruction must shift from predominantly instrumental approaches towards relational ones. We also identify key barriers in basic school contexts (e.g., time pressure, teacher beliefs, resource constraints) and propose a research agenda to support this shift.

***Keywords**: Skemp's cognitive development; relational understanding; instrumental understanding; basic school mathematics; mathematics education.*

Introduction

Mathematics at the basic school level is a foundational subject, not only for students' success in senior secondary education, but also for functioning in everyday life and in a rapidly evolving technological society. Yet numerous studies highlight that many learners in basic school (approximately ages 6-14) acquire mathematics superficially: they memorize procedures, apply them in familiar contexts, but struggle when confronted with unfamiliar situations, require transfer, or must reason "why" rather than simply "how". A key theoretical lens for understanding this phenomenon is provided by Skemp's (1976, 1987) distinction between instrumental understanding (knowing *how* to do something) and relational understanding (knowing both what to do and why). Since its introduction, this theory has influenced mathematics education research and practice, yet its implications for basic school mathematics teaching remain under-explored, especially in under-resourced and developing country contexts.

Richard Skemp saw cognitive development in mathematics as building interconnected "schemas" of ideas, where new concepts are attached to and reorganize prior knowledge. He argued that as these schemas grow more complex and connected, learners become able to reason flexibly, solve unfamiliar problems, and monitor and improve their own thinking ("intelligent learning").

Relational understanding means knowing both what to do in a mathematical task and why it works, so the learner grasps underlying structures and connections. This form of understanding supports transfer to new problems, easier recall, and long-term retention because knowledge is embedded in a well-connected schema rather than isolated rules.

Instrumental understanding is the ability to use rules and procedures to get correct answers without understanding the reasons behind them. It can give quick success on routine tasks but often leads to fragile knowledge that breaks down when problems change slightly or require explanation or justification.

Basic school mathematics usually refers to foundational topics such as arithmetic, elementary algebra, and basic geometry and statistics taught in primary and lower secondary school. In Skemp's terms, these topics can be learned instrumentally (following taught algorithms) or relationally (understanding number structures, operations, and relationships), with different consequences for students' cognitive development.

Mathematics education is the field concerned with how mathematics is taught and learned, including curriculum, teaching methods, learning theories, and assessment. Skemp's work is influential here because it highlights that valuing relational understanding and schema-building changes how teachers design tasks, explain concepts, and judge "success" in basic school mathematics.

Research across different contexts has shown that students often perform well on familiar procedural tasks but struggle to apply knowledge in novel contexts or explain the reasoning behind procedures (Lindenskov, 2023; Rachmawati, Subanti, & Usodo, 2022). Richard R. Skemp's distinction between *instrumental understanding* that is knowing rules and procedures without knowing why they work and *relational understanding* which is knowing both what to do and why offers valuable insight into these issues. At the basic school level, where foundational mathematical schemas are formed, fostering relational understanding is critical to developing long-term mathematical competence, transferability, and positive attitudes toward mathematics.

This paper revisits Skemp's theory and its cognitive development implications, then explores how these ideas apply to teaching and learning mathematics at the basic school level. We begin by articulating the theoretical framework, followed by a literature review of recent empirical studies applying or extending Skemp's constructs, then discuss practical implications for teaching, curriculum and assessment, and finally propose recommendations and further research directions.

Theoretical Framework: Instrumental and Relational Understanding

Skemp's original conceptualization

In his seminal paper "Relational Understanding and Instrumental Understanding" (1976), Skemp distinguished two qualitatively different modes of mathematical understanding. Instrumental understanding involves the ability to apply rules, procedures, algorithms and ensure correct answers, but without necessarily understanding why those rules work or how they relate to other concepts. Relational understanding, by contrast, involves comprehending how and why mathematical ideas work, making connections between them, and thereby enabling flexible application and transfer (Skemp, 1976; see also Skemp, 1987).

Skemp argued that although instrumental understanding can lead to correct answers in familiar tasks and is easier to achieve (especially in time-pressured classroom contexts), it is inferior in the long-term because it lacks conceptual networks, limits transferability, and tends to be fragile when tasks change or contexts vary (Skemp, 1976). A more recent theoretical analysis by Lindenskov (2023) clarifies five key characteristics of instrumental vs relational understanding, and emphasizes how these models can serve as analytical frameworks for educational research.

Cognitive development implications

Skemp's distinction aligns with cognitive learning theories: relational understanding corresponds to schema formation, conceptual networks, meaning-making and higher-order thinking; instrumental understanding corresponds to procedural fluency, rule memorization, and lower cognitive engagement. For example, Pathmanathan (2023) positions Skemp within a cognitive framework and argues that relational understanding supports better transfer and flexible problem solving.

In addition, Skemp (1987) argued that relational understanding supports better retention: because the learner understands the underlying relations, the knowledge is easier to recall, adapt, and extend. This is particularly important in mathematics where new topics build on old ones, such as arithmetic → algebra → geometry.

Relations to other theoretical perspectives

Recent studies reaffirm these distinctions in modern contexts, such as through latent profiles of conceptual-procedural knowledge (Lenz et al., 2024) and interventions emphasizing both types for better performance (Ncube, 2025).

Progressivism-based instruction and concept-based teaching continue to highlight conceptual depth over rote methods, with bi-directional links between knowledge types noted in reviews (Rittle-Johnson & Schneider, 2015).

The question of transfer is a special challenge in mathematics teaching because many curricula have fostered an instrumental understanding, which makes transfer difficult for the students.

Skemp's model complements these views by emphasizing the "what to do/why" distinction and placing it in a classroom/teacher discourse context: the rationales for learning (I-rationale and S-rationale) that underlie instrumental or relational orientations (Mellin-Olsen & Skemp, as discussed in Lindenskov, 2023).

Application to Basic School Mathematics

Why basic school mathematics matters: Basic school that is often primary to lower secondary is the stage where foundational mathematical concepts such as number operations, fractions, measurement, geometry, early algebraic thinking are introduced and developed. The quality of understanding at this stage has long-term implications for learners' mathematical trajectories, attitudes, retention and transfer. If students develop primarily instrumental understanding at basic school, they may struggle when topics become more abstract, require reasoning, or connect across domains

Evidence of instrumental vs relational understanding in basic school contexts: Recent empirical work has applied Skemp's framework in various contexts. For example, Rachmawati, Subanti and Usodo (2022) described students in grade 7 solving set-theory problems, showing that students with so-called "rational personality types" tended to exhibit relational understanding, being able to explain reasons, apply multiple strategies, rewrite problems, and represent them in images or symbols.

A study on exponential problems (Hidaiyah, Sukoriyanto & Slamet, 2024) found students with high ability showed relational understanding that is finding relationships, explaining why, while those of moderate ability showed instrumental understanding (applying memorised procedures) and those of low ability showed neither clearly.

Kuncorowati, Mardiyana & Saputro (2018) in the quadrilateral topic found students' main difficulty was "relating various mathematical concepts," an indicator of weak relational understanding.

Andam, Awuah and Obeng-Denteh (2025) investigated Grade 11 students' understanding of probability concepts via Skemp's framework. Although not basic school level, the study found many students relied on instrumental understanding and struggled to explain or adapt procedures.

These findings suggest that although relational understanding is possible even for younger learners, many are stuck in instrumental modes, especially in teachers and classroom contexts that emphasize procedure, factual recall, rapid coverage, and exam performance.

Implications of the distinction for basic school instruction: Skemp's distinction between instrumental and relational understanding guides basic school mathematics teaching by prioritizing conceptual grounding before procedural fluency, such as exploring part-whole relationships in fractions alongside procedures (Lenz et al., 2024).

Key Instructional Strategies

1. Tasks for relational understanding: Design tasks requiring explanations, multiple representations, connections across topics like number and geometry, and novel applications to foster deeper insight (Building

Algebraic Reasoning in Early Mathematics, 2025)

2. Teacher discourse and beliefs: Shift from rule-following ("here's the rule, do it") to relational ("here's the idea, why does it work?") through questioning, tasks, and feedback that promote exploration.
3. Assessment alignment: Include reasoning, explanations, novel tasks, and multiple representations to avoid skewing toward instrumental methods (Sibiya, & Essien, 2025).
4. Sequencing and time allocation: Allocate time for exploration, discussion, reflection, and consolidation despite curriculum pressures, rather than superficial coverage.
5. Manipulatives and representations: Use visual, verbal, physical, and symbolic tools, especially for younger learners in geometry, measurement, and arithmetic, to link "why" to "how" (Lanigan, 2025).

Challenges for Implementation in Basic School Contexts

While the theoretical benefits of relational understanding are clear, real-world implementation in basic schools faces multiple challenges. Many basic school mathematics teachers were taught instrumentally through rote, drill-based methods and may default to similar practices; changing this requires sustained professional development, reflective practice, and supportive pedagogical culture (Pugoy et al 2020). A recent study on out-of-field mathematics teachers found that teacher understanding, including pedagogical understanding for mathematics (PUFM), and relational orientation correlated positively with better teaching quality (Ni Riordáin & Hannigan, 2022).

In many jurisdictions, especially developing countries, teachers face pressure to cover many topics, prepare students for external exams, and ensure high pass rates, which leads to instruction focused on instrumental methods such as procedural drills instead of deeper exploration (Bolton, 2019). Instrumental understanding tends to dominate when curricula emphasize quantity over depth.

Resource constraints are common in basic schools, where lack of manipulatives, technology, and support for differentiated tasks means relational teaching demands more planning and resources than are available (Tibane, 2025). This limits effective relational mathematics teaching, impacting both teacher practices and student engagement (Tibane, 2025; Ni Riordáin & Hannigan, 2022).

Assessment practices that emphasize routine procedural questions reinforce instrumental teaching approaches and misalign with curriculum goals of reasoning and critical thinking (Bolton, 2019). This assessment misalignment leads to surface learning and poor motivation, as assessments fail to measure deeper relational understanding (Bolton, 2019; Ni Riordáin & Hannigan, 2022).

Learner diversity and readiness also pose challenges; learners come with varied backgrounds and levels of readiness for abstract reasoning, requiring procedural scaffolding alongside relational teaching, which further increases demands on teachers' time and expertise (Tibane, 2025). Challenges are highlighted as follow:

1. Teacher preparation and beliefs: Many basic school mathematics teachers themselves were taught instrumentally, such as rote, drill-based methods and may default to similar practices. Changing this orientation requires sustained professional development, reflective practice and supportive pedagogical culture. The recent study on out-of-field mathematics teachers found teacher understanding (PUFM) and relational orientation correlated with better teaching.

2. Curriculum load and examination pressure: In many jurisdictions, including developing countries, teachers face pressure to cover many topics, prepare students for external examinations and ensure high pass rates. This can push instruction toward instrumentality: fast coverage, procedural drills, and minimal exploration. As noted in the literature, instrumental understanding tends to dominate when curricula emphasize quantity over depth.

3. Resource constraints: Basic schools often lack manipulatives, technology, differentiated tasks or small-group support. Relational tasks may require additional planning and resources which may be scarce in under-resourced settings.

4. Assessment practices: If assessments continue to emphasize routine procedural questions, teachers will focus on instrumental teaching. Studies show weak alignment between curricula aims (critical thinking, reasoning) and assessment formats.

5. Learner diversity and readiness: In basic school contexts, learners come with varied backgrounds, prior knowledge, and readiness for abstract reasoning. While relational understanding is desirable, some learners may still require strong procedural scaffolding. Teachers must differentiate and scaffold accordingly, but this further increases demands on teacher time and expertise.

Suggestions for Practice

Sustained continuous professional development (CPD) enables teachers to differentiate instrumental from relational understanding and redesign tasks for conceptual depth (Alfred et al., 2023). Professional learning communities support sharing relational tasks and reflecting on student responses, with coaching emphasizing relational over procedural approaches (Zehetmeier et al., 2021).

Curricula should feature fewer topics for deeper exploration, integrating concrete, pictorial, symbolic, and verbal representations to foster relational webs (Skemp, 1976). Tasks prompting explanations of rules, multiple representations, or novel applications promote relational understanding in primary learners. This approach aligns with primary mathematics frameworks emphasizing connection-making.

Assessments blend procedural fluency items with reasoning tasks for explanation, connection, and transfer to evaluate relational understanding (Kanjee, 2020). Formative practices like interviewing students and providing "why/how" feedback address misconceptions effectively. These methods particularly benefit low-achieving primary students by linking formative to summative outcomes.

Teachers unpack procedural "why"s, model relational links to prior knowledge, and encourage peer explanations with representation shifts (Guillermo, 2025). Manipulatives aid visualization in geometry, fractions, and number sense, balancing fluency after conceptual work (Carbonneau et al., 2013). Recent studies confirm manipulatives improve attitudes and achievement in grade one pupils (Guillermo, 2025).

Basic schools require resources like manipulatives and planning time, especially in under-resourced Nigerian contexts with large classes (NFER, 2017). Policies must shift evaluations toward conceptual depth over coverage and exam scores (Awofala, 2017).

Longitudinal studies tracking relational development predict retention and transfer in basic schools (Webster et al., 2018). Interventions comparing relational to instrumental instruction, teacher beliefs, and resource constraints in Nigeria need expansion (Awofala, 2017). Cross-cultural validation of relational assessment tools for ages 6-14 fills key gaps (Kanjee, 2020).

Based on the foregoing analysis, the following recommendations emerge for teaching mathematics in the basic school level with a view to promoting relational understanding. These can further be breaking down as follow:

Professional Development for Teachers

1. Teachers should offer sustained continuous professional development (CPD) that helps teachers recognize the difference between instrumental and relational understanding, reflect on their practice, and redesign tasks and assessments accordingly.
2. Government should create professional learning communities where teachers share relational tasks, reflect on student responses, and adjust practice.
3. Teacher should include classroom-based coaching and mentoring focusing on relational pedagogy rather than merely procedural drills.

Curriculum and Task Design

1. Review curricula to adjust for fewer topics with greater depth, allowing time for exploration, discussion and connection-making rather than rushing through many procedures.
2. Develop and share tasks that promote relational understanding: e.g., “Explain why this rule works”, “Represent this concept in three ways”, “Apply this idea in a new context.”
3. Incorporate multiple representations such as concrete, pictorial, symbolic, verbal into tasks so learners build relational webs.

Assessment Reform

1. Design assessments with a mix of items: routine procedural tasks for fluency, and reasoning tasks for

relational understanding that will include explain, connect, and transfer.
2. Use formative assessment practices: collect student reasoning, interview or scaffold misunderstandings, provide feedback focused on "why" and "how".
3. Monitor student responses for evidence of relational understanding: e.g., ability to transfer procedures, explain why, and relate concepts.

Classroom Practice

1. Teachers should explicitly unpack "why" behind rules/procedures and model relational reasoning. "Here's the rule, but why does it work? How does it link to what we've done before?".
2. Encourage student talk, explanation, peer discussion, representation changes (drawing, objects, and symbols).
3. Use manipulatives and visualization especially at basic school level: for geometry, measurement, number sense, fractions.
4. Balance fluency and meaning: it's important that learners build fluency, but not at the cost of meaning. Procedural practice should follow conceptual exploration, not substitute it.

Resource and Policy Support

1. At the school and education system level, there should be allocation of resources especially to basic schools for manipulatives, visual representations, teacher time for planning and collaboration.
2. Policy makers should adjust curricular demands, assessment formats and teacher evaluation to value relational understanding and conceptual depth, not just coverage and exam scores.

Future Research Directions

While the body of research applying Skemp's framework is growing, especially at higher levels of mathematics education, there is still a need for more research at the basic school level, especially in diverse contexts that is developing countries, under-resourced schools like Nigeria. Possible directions include:

1. Longitudinal studies tracking students' development of relational vs instrumental understanding over time in basic school and how that predicts later performance, retention and transfer.
2. Intervention studies in basic school settings: designing and evaluating

tasks/instruction aimed at promoting relational understanding, comparing to instrumental-oriented instruction.

3. Research on teacher beliefs, classroom discourse, and institutional constraints that support or inhibit relational understanding in basic school mathematics.
4. Investigation into how resource constraints (large classes, limited manipulatives, teacher workload) mediate the implementation of relational pedagogy in basic schools.
5. Cross-cultural studies: how relational understanding manifests in various cultural/educational contexts, and how Skemp's distinctions might be adapted or extended for different educational systems.
6. Development of assessment instruments specifically for relational understanding in basic school mathematics (ages 6-14) and validation of such instruments in diverse contexts.

Conclusion

The distinction between instrumental and relational understanding, introduced by Skemp, continues to provide a rich and highly relevant theoretical lens for mathematics education, particularly at the basic school level. In this foundational stage of mathematics learning, the quality of understanding matters: students who learn relationally are better positioned for flexible thinking, transfer, retention and future success. Yet the prevailing teaching practices in many basic school contexts lean heavily instrumentally, due to time constraints, curriculum demands, resource shortages and teacher beliefs. The shift toward relational understanding requires systemic changes: teacher professional development, curriculum redesign, assessment reform, classroom practice changes and resource support. Entrenching relational thinking in basic school mathematics is not easy, but given its long-term benefits for learners, it is necessary. Future research focusing on the basic school level will deepen our understanding of how to support such a shift in diverse educational contexts.

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